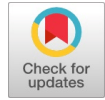




The Classical Electron Radius, the Electron Compton Wavelength and the Running Fine Structure Constant



Paolo Christillin

Abstract: *The fine structure constant variation as a function of the electromagnetic interaction strength is investigated and shown to evolve from the canonical standard 1/137 to 1 at distances of the order of the classical electron radius, i.e. at a typical strong interaction value. This confirms the outcome at Planck scales, but it happens even later in the Universe's evolution, closer to the present time, although it will also be experimentally unattainable. The result is simply derived by imposing quantum effects, i.e. the Compton wavelength at the classical electron radius, in analogy with the procedure by which the corresponding Compton wavelength constrains the Schwarzschild radius.*

Keywords: *Compton Wavelength, Classical Electron Radius*

It is perhaps worth stressing that, whereas for the hydrogen atom QM (i.e. the uncertainty principle) works to cure e.m. divergences, here the opposite happens in the sense that the classical radius is much smaller than the Compton wavelength $O(10^{-13}m)$.

This probably unexpected result backs up, however, what had been obtained by other methods for the coupling constants which unify at the Planck scale and can also be derived by considering that the Planck electric charge is [1]

$$q_p \simeq 11.7 e \quad (5)$$

which implies, by simply squaring the previous expression, that for Planck coupling constants

$$a_p = 1 \quad (6)$$

Therefore, one obtains that the "ordinary" fine structure constant evolves from the standard value 1/137 to 1

$$\alpha \rightarrow \alpha_{r_0} = 1 = \alpha_p \quad (7)$$

I. INTRODUCTION

The classical electron radius r_0 is obtained, as is well known, by imposing that its electrostatic energy cancels its relativistic energy.

$$mc^2 - \frac{e^2}{4\pi\epsilon_0 r_0} = 0 \quad (1)$$

i.e.

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} \quad (2)$$

Where e stands for the electric charge $e \simeq 1.6 \times 10^{-19}C$ and m for the electron mass $\simeq 10^{-30}kg$.

II. RESULTS

An additional constraint on this purely classical result arises from the Compton wavelength.

$$\lambda_e = \frac{\hbar}{mc} \quad (3)$$

and by equating the two last expressions. Thus, quantum effects imply

$$\frac{e^2}{4\pi\epsilon_0 hc} = \alpha_{r_0} = 1 \quad (4)$$

i.e. that the fine structure constant at $r_0 \simeq 10^{-15} m$ is one.

III. CONCLUSIONS

Thus, classical mechanics and QM unify at $\alpha=1$. The result might be criticized mainly because it is not stuffed with superfluous mathematical formalism but is based on the same simple theoretical arguments which combine the Schwarzschild radius and the particle wavelength, leading to the prediction of Planck units and the unification of the different coupling constants in that limit.

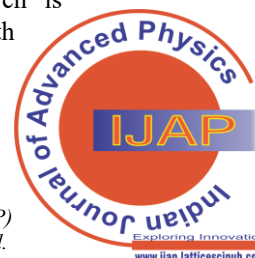
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DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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Paolo Christillin, Student from 1964 at the Physics Faculty of the University of Pisa as a pupil of the Scuola Normale Superiore. Graduated in 1968 and became an Associate Professor in 1982. Visiting professor at Niels Bohr Institute, CERN and Orsay. Main interests: photoreactions and gravitation. The results of the first line of research are summarised in the Physics Report article "Nuclear photoreactions at intermediate energies" (1990). As regards gravitation, he is the author of the book " Gravitation and cosmology for the simple mind(ed)" (2025), Aracne Ed, where all aspects of gravitation are considered; the present popular treatment is critically examined in particular as regards the reality of space deformation and matter creation necessary to account for the time-dependent age of the Universe.

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