

Redefinition of De-Broglie Wavelength Associated with Material Particle



Chandra Bahadur Khadka

Abstract: The objective of the work is to reveal the Association of special theory of relativity with De-Broglie wavelength of a particle. It shows that wave nature of a particle is always associated with its relativistic properties. The phenomena of interference, diffraction and polarization of light can only be explained on the basis of wave theory of light. These phenomena show that light possesses wave nature. However, certain phenomena like photoelectric effect, Compton effect and discrete emission and absorption of radiation can only be explained on the basis of quantum theory. According to this theory, light propagates in a small packets or quanta and behave like corpuscles or particle. This shows that light possesses particle nature. thus, we can say that light possesses dual nature. The matter also possesses the same dual nature. In order to have wave nature, the particle must satisfy the equation $\lambda = \frac{h}{mv}$ Where

m denotes the mass of particle when it is moving with certain velocity. $h=6.62 \times 10^{-34}$ is the plank's constant. C is the velocity of light in free space. This shows that wavelength of a material Particle depends on mass. When mathematical operation is done on above equation, it give complete quantitative properties about the wave nature of a particle.

Keywords: De -Broglie wav length, Dual nature, Quantum theory, Relativity

I. INTRODUCTION

De-Broglie in 1923-24, suggested that material particles such as electron, proton etc. should also possess dual nature like radiation. Consider photon of frequency f , Then energy of photon is $E = hf$ where $h = 6.62 \times 10^{-34}$ is the plank's constant. Let m be the equivalent mass of the photonic energy. Then from Einstein's mass energy relation.

$$E = mc^2 = hf$$

$$\text{Or, } m = \frac{hf}{c^2}$$

$$\text{Or, } mc = \frac{hf}{c}$$

Put $mc = p$ (momentum of light) and

$$\lambda = \frac{c}{f} \text{ then}$$

$$p = \frac{h}{\lambda}$$

Where λ is the wavelength of radiation.

If m is the mass and v is the velocity of the moving particle, then momentum $P = mv$ and the wavelength of waves associated with material particle is given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Which is De-Broglie wavelength for particle.

Momentum of Photon (mc) is replaced by momentum of particle (mv) in above process of deriving De-Broglie wavelength of a particle, as consequence, converse process contradicts Einstein mass energy relation (mc^2) as follows

$$\lambda = \frac{h}{mv}$$

$$mv = \frac{h}{\lambda}$$

Multiplying both sides by v , we get

$$mv^2 = \frac{hv}{\lambda}$$

$$mv^2 = hf$$

Where $f = \frac{v}{\lambda}$ be frequency of moving material particle.

So, Einstein mass energy associated with particle at certain Frequency f is mv^2 instead of mc^2 . Thus, the process of deriving De-Broglie wavelength is completely wrong.

II. METHODS

A. Proof of De-Broglie wavelength

let a particle of initial frequency f is moving away from gravitational field of star of mass M and the potential energy of a particle of mass m at distance R from surface of star is

$$E_p = -\frac{GMm}{R}$$

Negative sign indicated that the force between two masses is attractive.

Consider a particle is moving at velocity of light c of equivalent mass m and frequency f on the surface of star. The potential energy of the particle on the surface is

$$-\frac{GMm}{R} = -\frac{GMhf}{Rc^2} \left(\because mc^2 = hf \right)$$

$$\therefore m = \frac{hf}{c^2}$$

Total energy of the particle is

$$E = \text{Quantum energy} + \text{potential energy}$$

$$= hf - \frac{GMhf}{Rc^2}$$

$$= hf \left(1 - \frac{GM}{Rc^2} \right)$$

The frequency of particle should be change due to addition of potential energy .so f' is the new frequency of particle

So, $E = hf'$

$$hf' = hf \left(1 - \frac{GM}{Rc^2} \right)$$

$$\frac{f'}{f} = 1 - \frac{GM}{Rc^2}$$

$$\text{or, } 1 - \frac{f'}{f} = \frac{GM}{Rc^2} \quad (1)$$

Manuscript received on 29 March 2022 | Revised Manuscript received on 05 April 2022 | Manuscript Accepted on 15 April 2022 | Manuscript published on 30 April 2022.

* Correspondence Author

Chandra Bahadur Khadka*, Student of Trichandra Multiple Campus, Tribhuvan University, Kathmandu, Nepal. Email: chandrabahadur9988@gmail.com

© The Authors. Published by Lattice Science Publication (LSP). This is an open access article under the CC-BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)



$$\text{or, } \frac{f-f'}{f} = \frac{GM}{RC^2}$$

$$\text{or, } \frac{df}{f} = \frac{GM}{RC^2}$$

Which is relative frequency change.

If $\frac{GM}{RC^2} \geq 1$ it is clear no particle leave the star and this star is called black hole.

If for black hole

$$\frac{GM}{RC^2} \geq 1$$

$$\frac{df}{f} \geq 1 \quad (2)$$

Now, consider initially a particle of mass m move around the star at velocity of light (c) due to gravitation force then it's centripetal force is $\frac{mc^2}{R}$

Where R is distance between center of star and particle.

Let M amount of mass of star is deleted. Then gravitational force on particle is change by

$$\frac{GMm}{R^2}$$

Then, Velocity of particle get change as result mass should also varies. let m_1 and V be new mass and Velocity of particle

New centripetal force

= initial centripetal force – decrease in gravitation force

$$\frac{m_1 v^2}{R} = \frac{mc^2}{R} - \frac{GMm}{R^2}$$

Dividing both sides by $\frac{mc^2}{R}$

$$\frac{m_1 v^2}{mc^2} = 1 - \frac{GM}{RC^2}$$

$$\text{Or, } \frac{m_1 v^2}{mc^2} = 1 - \left(1 - \frac{f'}{f}\right) \quad \text{using Equation (1)}$$

$$\text{Or, } \frac{m_1 v^2}{mc^2} = \frac{f'}{f}$$

$$\text{Or, } \frac{m_1 v^2}{f'^2} = \frac{f}{f}$$

We have

$$mc^2 = hf$$

$$\frac{mc^2}{f} = h$$

Then

$$\frac{m_1 v^2}{f'} = \frac{mc^2}{f}$$

$$\text{Or, } \frac{m_1 v^2}{f'} = h$$

$$\text{Or, } m_1 v^2 = hf'$$

$$\text{Or, } m_1 v^2 = \frac{hv}{\lambda'}$$

$$\text{Or, } m_1 v = \frac{h}{\lambda'}$$

$$\lambda' = \frac{h}{m_1 v}$$

This is known as De- Broglie Wavelength

B. Relativity and wave nature of particle

De – Broglie wavelength of a particle is

$$\lambda = \frac{h}{mv}$$

$$\text{Or, } v = \frac{h}{\lambda m}$$

According to Special relativity

$$M = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Put } v = \frac{h}{\lambda m}$$

$$\text{Or, } m = \frac{m_0}{\sqrt{1 - \frac{h^2}{m^2 \lambda^2 c^2}}}$$

$$\text{Or, } m \left(\sqrt{m^2 \lambda^2 c^2 - h^2} \right) = m_0 m \lambda c$$

Squaring both sides

$$\text{Or, } m^2 (m^2 \lambda^2 c^2 - h^2) = m_0^2 m^2 \lambda^2 c^2$$

$$\text{Or, } m^2 \lambda^2 c^2 - h^2 = m_0^2 \lambda^2 c^2$$

$$\text{Or, } m^2 \lambda^2 c^2 - m_0^2 \lambda^2 c^2 = h^2$$

$$(\lambda c)^2 (m^2 - m_0^2) = h^2$$

$$(\lambda c)^2 = \frac{h^2}{m^2 - m_0^2}$$

$$\lambda = \frac{h}{c \sqrt{m^2 - m_0^2}}$$

Now, in reverse process

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Or, } m^2 \left(1 - \frac{v^2}{c^2}\right) = m_0^2$$

$$\text{Or, } m^2 - \frac{m^2 v^2}{c^2} = m_0^2$$

$$\text{Or, } m^2 - m_0^2 = \frac{m^2 v^2}{c^2}$$

$$\text{Or, } \sqrt{m^2 - m_0^2} = \frac{mv}{c}$$

$$\text{Or, } mv = c \sqrt{m^2 - m_0^2}$$

According to de-Broglie theorem

$$\lambda = \frac{h}{mv} = \frac{h}{c \sqrt{m^2 - m_0^2}}$$

$$\therefore mv = c \sqrt{m^2 - m_0^2}$$

$$\frac{h}{\lambda} = c \sqrt{m^2 - m_0^2}$$

$$\lambda = \frac{h}{c \sqrt{m^2 - m_0^2}}$$

i.e. wavelength of a particle exist whenever mass variation take place. de-Broglie wavelength exist in only relativistic

C. Experimental Section

In Division and Germer's experiment the wavelength of electron using de-Broglie hypothesis was 1.66 Å at 54v

Let electric field of potential V is applied to electron of charge e and mass m at velocity v . then

Applied potential Energy to electron = kinetic energy of electron

$$eV = \frac{1}{2} mv^2$$

$$\therefore 2 \frac{eV}{v} = mv$$

$$\text{Using de – Broglie hypothesis } mv = \frac{h}{\lambda}$$

$$\frac{h}{\lambda} = \frac{2eV}{v}$$

$$\therefore v = \frac{2eV \lambda}{h}$$

$$\text{Putting } e = 1.6 \times 10^{-19} \text{ C}, V = 54 \text{ V}, \lambda = 1.66 \text{ Å} \& h = 6.62 \times 10^{-34}$$

$$v = \frac{2 \times 1.6 \times 10^{-19} \times 54 \times 1.66 \times 10^{-10}}{6.62 \times 10^{-34}}$$

$$v = 4.34 \times 10^6 \text{ m/s}$$

Velocity of electron =

$$4.34 \times 10^6 \text{ m/s}$$



if $\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}}$ is true. Then it should give same value of wavelength
rest mass of electron (m_0) = 9.11×10^{-31} kg

mass of electron at velocity (v) = $4.34 \times 10^{6m/s}$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{9.11 \times 10^{-31}}{\sqrt{1 - \frac{(4.34 \times 10^{-31})^2}{(3 \times 10^8)^2}}} = 9.111 \times 10^{-31} \text{ kg}$$

$$\sqrt{m^2 - m_0^2} = 1.329317 \times 10^{-32}$$

$$\therefore \lambda = \frac{h}{c\sqrt{m^2 - m_0^2}} = \frac{6.62 \times 10^{-34}}{3 \times 10^8 \times 1.329317 \times 10^{-32}} = 1.66 \text{ \AA}$$

This shows that $m = \frac{h}{c\sqrt{m^2 - m_0^2}}$ is holds good.

III. RESULT AND DISCUSSION

A moving particle is associated with a wave and its wavelength is given by $\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}}$. It is clear that a matter particle will have wavelength associated with it only if the term $m^2 - m_0^2$ is non-zero. Since, wave nature of a particle exists due to variation of mass with velocity. variation mass occurs if and only if relativistic effect happens on nature. Wave properties of a particle exist whenever the condition is relativistic. So, wavelength of a material particle never exist in non-relativistic case. Thus every material particle behave wave nature in universe due to change of its mass with velocity.

Case A : When a particle is at rest then $m^2 - m_0^2 = 0$ then $\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}}$, $\lambda = \infty$ it is same result as given by $\lambda = \frac{h}{mv}$ at $v = 0$

Case B: When a particle is at velocity of light $m^2 - m_0^2 \sim m^2$ the $\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}} = \frac{h}{mc}$ it is same result as given by $\lambda = \frac{h}{mv}$ at $v = C$

Also, the experimental values are put in given equation of wavelength of particle data obtained from experimental study of mater waves: Davisson and Germer's experiment for experimental verification. This equation gives accurate value of wavelength of electron by calculation.

IV. CONCLUSION

De-Broglie wavelength in terms of mass of a particle is given by,

$$\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}}$$

$$\lambda \propto \frac{1}{\sqrt{m^2 - m_0^2}}$$

It reveals that wavelength of a material particle is inversely proportional to square root of difference of square of relative mass and square of rest mass. So, wavelength of a particle exist whenever $m^2 - m_0^2$ is not equal to zero. This condition

holds whenever Relativistic Phenomenon happen on nature. So wavelength of a particle exists whenever Relativity exist.

ACKNOWLEDGMENT

Author express his sincere Thanks to Dr. Bishma Karki lecture of Trichandra Multiple Campus, Tribhuvan University helps to submit on this journal.

REFERENCES

- Mackinnon, L. (1981). Particle rest mass and the De Broglie wave packet. *L ettere al Nuovo Cimento*, 31(2), 37-38. [CrossRef]
- Butto, N.(2020). A new Theory on Electron wave-particle Duality, *journal of high Energy physics, Gravitational and Cosmology*, 6(04) ,567 [CrossRef]
- Orozovic ,k., & Balon, B (2021). De Broglie wavelength Defined by the rest energy E(0). and its Velocity. *International Journal of Physical and Mathematical Science*, 15 (7) ,82-85
- Cha KraDhar ,i. ,& RagharaChanguui , i.v.v.(1993). Analogues of de Broglie relation and new quantum theory. *Indian journal of pure and applied Physics*, 31(3) , 141-147
- Alexy, R.(2010). The dual nature of law Ratio juris 23(2), 167-182 [CrossRef]
- Wang, X.(2021) . A Discussion about special Relativity International journal of scientific Research and modern Education, 6(2) ,4-7
- Selleri ,F. (Ed.).(1992). Wave particles duality (p.187). London: plenum press. [CrossRef]
- Dimitrova, T.L.& Weis ,A.(2008). The wave –particle duality of light: A demonstration Experiment. *American journal of physics*, 76(2), 137-142 [CrossRef]
- G Aruldas and P. Raja Gopal, *Modern Physics*, Prentice – Hall of India Private Limited, New Delhi -11001.
- J.D.Jackson .classical Electro dynamics, (third ed), Replika Press Pvt. Ltd. Kundli -131028
- S.N. Ghoshal, S. Chand and Company Ltd. *Atomic Physics* Ram Nagar, New Delhi, 110055
- Agrawal –Agrawal, *Fundamentals of modern physics*, PragatiPrakashan, Meerut -2500 (U.P)
- Arora C.L (1995), B.SC.physics, volume. I.I.S.Chand and Compaany Ltd. New Delhi.
- Dr. Gupta, Dr.Kumar Singh.(1992), *Electrodynamics (Electricity and Magnetism) A text book for B.SC(H ons.) and M.SC courses of all Indian Universities*, Pragatipraka Shan, Meerut up.
- A Dictionary of Physics(Fifth Edition)(2006), Oxford University Press.
- .B.L. Theraja *Modern physics*, S. Chand and Company Ltd. Ram Nagar, New Delhi -110055
- D.C. Tayal, *Nuclear Physics*, Himalya. Publishing house, Ramdoot, Dr. Bhalerao Marg. Bombay
- Kenneth S. Krane, *Modern Physics*, John Willey and Sons(sea) pte. Ltd. 37, Jalan Premimpin# 05-04 BLK B.Union Industrial Building Singapore 2057, Republic of Singapore
- S.O. pallai, *modern physics and solid state Physics*, New age International Publishers.
- WEBB, R.H (1969), *Elementary Wave optics*, Academic ppress, New York (USA)
- Young, H.D.(1968), *Fundamental of Optics and modern Physics*, McGraw –Hill Book Co. New York (USA)

AUTHORS PROFILE



Chandra Bahadur Khadka, is from Nepal. He presently live in Kathmandu, the capital city of Nepal. He is currently studying bachelor's degree in Physics from Trichandra Multiple Campus. Tribhuvan University Nepal. He has been writing various series of articles related to new theoretical discoveries in Physics

