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Mass Accretion During the Motion of Raindrops

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***Abstract:** Exploration of dynamics of raindrops is one of the simple yet most complicated mechanical problems. Mass accretion from moist air during the motion of raindrop through resistive medium holds an arbitrary power law equation. Its integral part is the change of shape, terminal motions and terminal solutions, etc. Classical Newtonian formalism is used to formulate a mathematical model of generalized first order differential equation. We have discussed about the terminal velocity of raindrop and its variation with the extensive use of python program and library. It is found that terminal velocity $v_T^{c\alpha\beta}$ is achieved within 20 seconds where $\alpha = (0, \frac{1}{3}, \frac{2}{3})$, $\beta = (0, 1)$ and $n = 0, 1, 2, 3, 4, \ldots$. Its variations due to mass accretion roughly follows the earlier predicted range $g/7$ to $g/3$.*

***Keywords:** Air Resistance, Mass Accretion Rate, Raindrop, Terminal Velocity.*

I. INTRODUCTION

Exploration of dynamics of raindrops is one of the simple yet most complicated mechanical problems studied since late 40s of last century. During its fall a raindrop attains a constant velocity, called the terminal velocity [1]. The earliest work is to determine terminal velocity of raindrops by electronic technique. An empirical study of the terminal velocities of falling raindrops for different drop sizes was presented by Gunn and Kinzer [2]. If v_T in meter/sec be the terminal velocity of a raindrop and R in meters be its radius (assuming raindrop to be spherical) its terminal velocity can be expressed as a function of its size and is given by $v_T = \sqrt{\frac{8gR(\rho_w - \rho_a)}{b\rho_a}}$. The droplets were taken very small to satisfy Stokes law and accuracy of the measurement was better than 0.7%. Due to air turbulence, water droplets in clouds collide, hence produce larger droplets. Thus raindrops vary widely in their shapes, sizes, velocities and have a wide size distribution. An empirical distribution for raindrop size was the Marshall and Palmer distribution [3]. Beard and Chuang [4] described the shape of a raindrop as a 10th order cosine distortion of a sphere. The shape of a raindrop was be described as a 10th order cosine distortion of a sphere by Beard and Chuang [4]. With increase in raindrop size, it becomes an oblate spheroid. Larger the size of raindrops, more are severely distortion, while smaller drops are almost spherical. The significant fraction of rain contains raindrops of less than 1mm in size and their shapes can be well approximated like a sphere. Their shapes will be governed by internal hydrostatic pressure, hydrodynamic pressure of medium and surface tension. A raindrop is axially symmetric along the line of motion. Measurement on rain at night in backscattered light was done by Beard et al [5]. Recently piezoelectric energy harvesting techniques is used to capture vibration, motion and acoustic noise of raindrops which is converted to electrical output. Recently vibration, motion and acoustic noise of raindrops which are converted to electrical output, are measured by piezoelectric energy harvesting techniques [6]. In recent years it may be an alternative energy source. The experimental results show a power output for one unit at around 2.5 mW. The kinematic behaviour of falling raindrop was deduced for a variety of mass accretion mechanisms and it was related to various closely related mechanisms. Assuming increase in mass of raindrop linearly with increasing distance and time, Krane [7] suggested a proportionality relation between mass accretion dm/dt, mass m and velocity vwith different powers of these two variables. Adawi [8] suggested $\frac{dm}{dt} = cm^{\alpha}v$ with $\alpha = 0, \frac{2}{3}$ for zero acceleration case. Partovi and Aston [9] took air resistance to this problem which is proportional to the square of velocity. Sokal1 [10] generalized Krane's idea by introducing $\frac{dm}{dt} = cm^{\alpha}v^{\beta}$ with easy case $(\alpha, \beta) = (\frac{2}{3}, 0)$ and difficult case $(\alpha, \beta) = (\frac{2}{3}, 1)$.

All these suggest a need to generalize the dynamics of raindrops. In this article this generalization and corresponding calculation is done.

II. FORMULATIONS

Raindrop problem is more interesting than rocket motion because when generalized, its motion through moist air in all directions (basically up and down) $\vec{F} = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v}$ is a complicated one. This force consists of downward weight of the raindrop $mg\hat{k}$, buoyant force $-m\rho_a/\rho_w\hat{k}$ (ρ_a and ρ_w are the density of the medium and raindrop respectively), and the resistive frictional force of the medium, which is proportional to the nth power of velocity $-bv^n\hat{k}$. Assuming $a = g(1 - \rho_a/\rho_w)$ the new raindrop equation is

$$m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v} = am\hat{k} - bv^n\hat{k} \quad (1)$$

Here b is defined as the resistive force per unit nth power of velocity. Lynch and Lommatsch [11] assumed the cross-sectional area of raindrop of the form $A = 3.3108 \times (2R)^{2.21672}$ and the mass of raindrop of the form $m = 957.251 \times (2R)^{3.09275}$ where R is the approximate radius of raindrop.

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Finally they took the value of resistive constant as $b = 0.15\rho_a A$. Generally, for resistive force proportional to velocity we put n=1. But generalized procedure for a solution is to be taken in those cases for the examples where n>1. In case of mass accretion in moist air the raindrop collects mass and rate of mass accretion may be written as $\frac{dm}{dt} = \rho_a vA$. Here $A = \pi R^2$ is the projected area of the raindrop and it is the largest cross section. For spherical raindrop, the increase in radius is proportional to $m^{1/3}$ and the increase in area is proportional to $m^{2/3}$. Generalization yields $\frac{dm}{dt} = cm^{\alpha}v^{\beta}$ where $c > 0$ is a constant and α and β are exponents with values earlier mentioned [10]. If the actual shape of the raindrop is spherical then the value of the constant c can be written as [9]

$$c = \frac{\pi R^2}{(\frac{4}{3}\pi R^3 \rho_w)^{\alpha}} \quad (2)$$

III. RESULTS AND DISCUSSION

Analytical solutions are complicated [7-10] and computational solutions are comparatively easier. No mass accretion case of equation (1) was solved analytically and by using python code [12]. It has been shown that there is impact of diameter and mass of raindrop on terminal velocity and the general solution was

$$\sum_{l=1}^{n} s_l \ln\frac{r_l}{r_l - v} = \frac{bt}{m} \quad (3)$$

There a python code has been developed and they showed that the terminal velocity is reached almost within ten seconds for mass of raindrop 0.5 mg and approximate diameter 1 mm. The minimum raindrop size below which the raindrop may be assumed to be a cloud particle is of mass 10^{-06} mg.

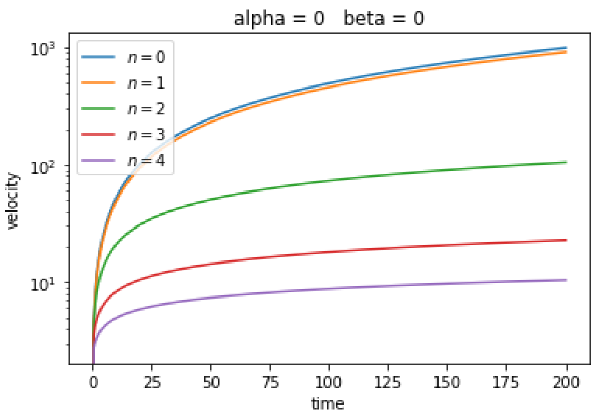

Figure 1 Velocity-time graph in SI units for $(\alpha, \beta) = (0, 0)$

General solution for mass accretion is rather more complicated. For lower values of α, β and n we have done both analytical and computational results for terminal velocity $v_T^{c\alpha\beta}$ which is

$$cv_T^{c00} + b(v_T^{c00})^n = aM \quad (4)$$

For example when $\alpha = 0$, $\beta = 0$, $c \neq 0$ and $n = 0$ we have $\frac{dv}{dt} = 0$ and therefore $m = M$ and $M = (\frac{2bm_0 - am_0^2}{a})^{1/2}$. Here m_0 is the mass of raindrop at time $t = 0$.

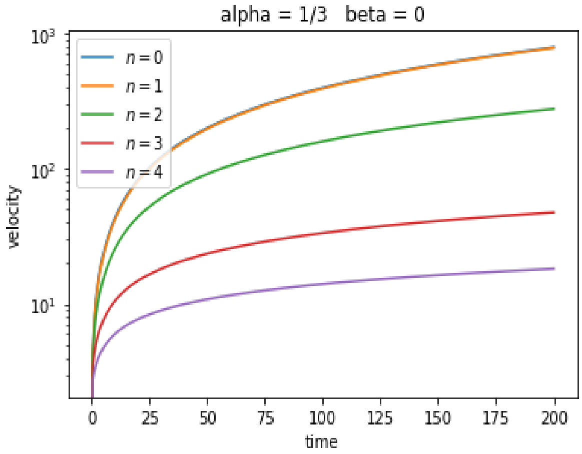

Figure 2 Velocity-time graph in SI units for $(\alpha, \beta) = (\frac{1}{3}, 0)$

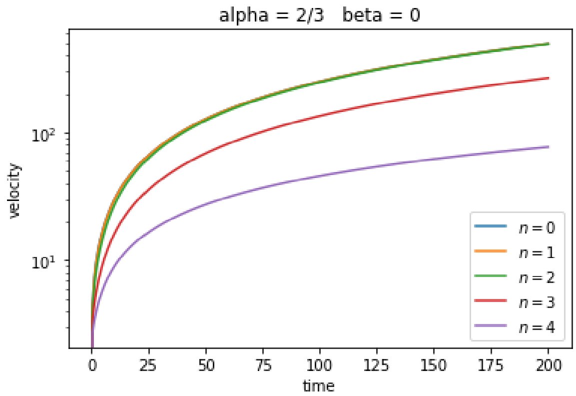

Figure 3 Velocity-time graph in SI units for $(\alpha, \beta) = (\frac{2}{3}, 0)$

Table 1 Input parameters

Air density ρ_a in kg/m^3	0.006211*
Water density ρ_w kg/m^3	957.251*
Acceleration due to gravity g in m/s^2	9.81*
Radius R in m	0.00001– 0.01
$b = 0.15\rho_a A$ *	$4.91\times10^{-12} - 6.84\times10^{-5}$
$a = g(1 - \rho_a/\rho_w)$	9.8099363

* Ref. [11]

Similar expressions for $\alpha = \frac{1}{3}$, $\beta = 0$, $c \neq 0$ and $\alpha = \frac{2}{3}$, $\beta = 0$, $c \neq 0$ are

$$cv_T^{c\frac{1}{3}0} M^{\frac{1}{3}} = aM - b(v_T^{c\frac{1}{3}0})^n \quad (5)$$

$$cv_T^{c\frac{2}{3}0} M^{\frac{2}{3}} = aM - b(v_T^{c\frac{2}{3}0})^n \quad (6)$$

Using Table 1 for input parameters the velocity versus time plot of all the graphs of figure 1, 2 and 3 for $\alpha = 0, \frac{1}{3}, \frac{2}{3}$ and $\beta = 0$ are almost similar to no mass accretion case [12]. Due to mass accretion terminal velocity $v_T^{c\alpha 0}$ is not perfectly horizontal to time axis ranging from approximately 1000 m/s for lowest value of $n = 0$ to 20 m/s for $n = 4$.

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The graphs show terminal velocity is achieved within 20 seconds. After that due to mass accretion it increases. More the power of dissipation with velocity less is the increase in mass. $n = 0$ and $n = 1$ case are very close to each other indicating that they are almost independent of retardation. More and more terminal velocity values $v_T^{c\alpha 0}$ for lower n converge. Also these values increase with value of α.

Table 2 Slope of variation of terminal velocity (SI units) due to mass accretion

	$n \rightarrow$	0	1	2	3	4
(α, β)	(0,0)	4.899	4.527	0.194	0.024	0.008
(α, β)	(1/3,0)	3.919	3.907	0.949	0.086	0.022
(α, β)	(2/3,0)	2.450	2.450	2.450	1.319	0.245
(α, β)	(0,1)	3.266	3.266	2.944	0.158	0.022
(α, β)	(1/3,1)	2.450	2.450	2.449	1.968	0.199
(α, β)	(2/3,1)	1.400	1.400	1.400	1.400	1.400

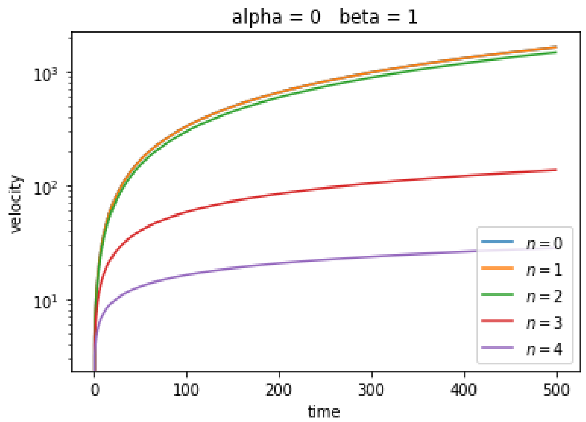

Figure 4 Velocity-time graph in SI units for $(\alpha, \beta) = (0, 1)$

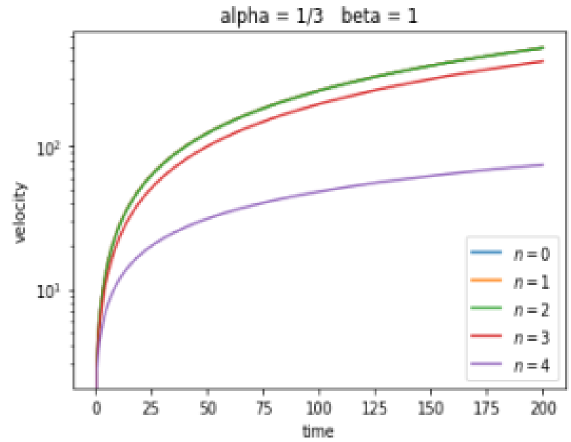

Figure 5 Velocity-time graph in SI units for $(\alpha, \beta) = (\frac{1}{3}, 1)$

Similar expressions like equations (5) and (6) are obtained when $c \neq 0$, $\alpha = 0, \frac{1}{3}, \frac{2}{3}$ and $\beta = 1$ which are

$$cv_T^{c01} = aM - b(v_T^{c01})^n$$

$$cv_T^{c\frac{1}{3}1} = aM - b(v_T^{c\frac{1}{3}1})^n$$

$$cv_T^{c\frac{2}{3}1} = aM - b(v_T^{c\frac{2}{3}1})^n$$

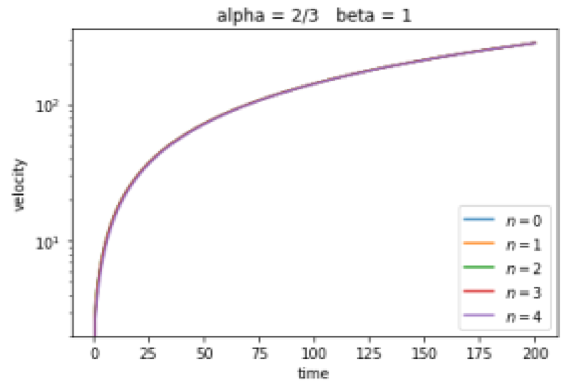

Figure 6 Velocity-time graph in SI units for $(\alpha, \beta) = (\frac{2}{3}, 1)$

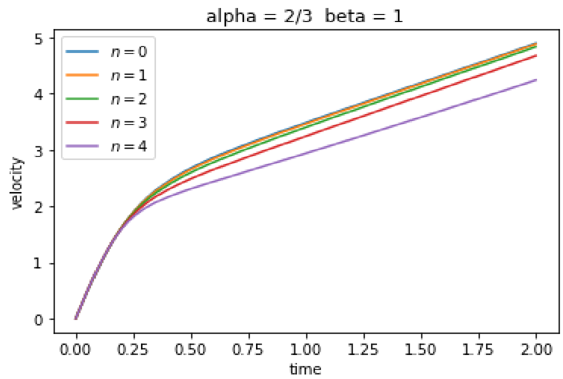

Figure 7 Velocity-time graph in SI units for $(\alpha, \beta) = (\frac{2}{3}, 1)$

We have almost similar graphs of figure 4, 5 and 6 in these three cases. The exception is that all terminal velocity $v_T^{c\alpha 1}$ values converge to a single curve and with β it increases. Figure 7 shows the slight variation of velocity of figure 6 although they converge.

All these seven plots clearly indicate that terminal velocity $v_T^{c\alpha\beta}$ gradually decreases with increasing n to a single value for $\alpha = (0, \frac{1}{3}, \frac{2}{3})$, $\beta = (0,1)$. With the increase in the value of (α, β) the splitting of the graphs for $n = 0,1,2,3,4, \ldots$ decreases with increase in time. The slope of these graphs also decreases except the last case of figure 6 which is indicated in Table 2. Krane [7] pointed out that acceleration takes the form $\frac{g}{k}$ where k is an integer larger than unity and more specifically it takes the form $\frac{g}{7}$ to $\frac{g}{3}$. Table 2 reflects some matching with this range. It is clear from the formulation as well as from table 2 that mass accretion change terminal velocity $v_T^{c\alpha\beta}$ and that change decreases with the increase of n. Computer programming using library of python code for solving simultaneous equations of mass accretion is taken into account.

IV. CONCLUSION

Here the general first order differential equation for mass accretion of raindrop $\mathbf{m\frac{dv}{dt} + cm^{\alpha}v^{\beta+1} = am - bv^n}$ is solved using python program and library to achieve terminal velocity $\mathbf{v_T^{c\alpha\beta}}$.

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More the indices **n** of velocity dissipation less is the increase in mass during the process. Also these values increase with value of $\boldsymbol{\alpha}$. With the increase in the value of $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ the splitting of the graphs decreases with increase in time. Here the first order ordinary differential equation of raindrop is solved analytically and using python program. This will best suit for the developing undergraduate theoretical knowledge and the novel ideas.

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APPENDIX

[Python Code]

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
#some initial values
d = 0.001
R = d/2
c = 0.3
pw = 1000
pa = 1.161 # at 300K, 1 bar
A = 3.3108*(d**2.21672)
m0 = 1000 * 0.957251 * (d**3.09275) #initial
mass value
v0= 0
t0 = 0
g = 9.81
p = g * (1-(pa/pw) )
q = 1/2 * c*pa*A
sol = [v0]
t= np.linspace(t0,200,600)
def rain(u,t,n,a,b):
ca = (np.pi * R**2)/(4/3 * np.pi * R**3 *
pw)**a
m , v = u[0] , u[1]
dmdt =  ca * m**(a)  * v**(b)
dvdt = p - ((q*(v**n))/m) - ca * m**(a-1)
*v**(b+1)
return [dmdt , dvdt]
for n in range(0,5):
u0 = [m0,v0]
sol = odeint(rain,u0,t,args=(n,2/3,1))
m , v = sol[:,0] , sol[:,1]
plt.plot(t, v)
plt.legend(['$n=0$','$n=1$','$n=2$','$n=
3$','$n=4$','$n=5$'])
plt.title ('alpha = 2/3  beta = 1')
plt.xlabel('time')
#plt.yscale('log')
plt.ylabel('velocity')
plt.show()
```

AUTHORS PROFILE

Sneha Dey, is an undergraduate Physics Honours student of Maulana Azad College, Kol-13. She attended an workshop 'Inspire Camp' organized and sponsored by The Department of Science and Technology, Central Government of India (DST) and DBT held at Jagadis Bose National Science Talent Search, Kolkata, West Bengal, India. She presented a paper and ppt on mechanical properties of matter after working in the workshop. She also attended an experimental workshop named 'C. K. Mazumdar Memorial Workshop on Experimental Physics' organized by All India Physics Teachers Association. She worked on making of a "LiFi (Light Fidelity)" and presented the project in a science exhibition organized by St. Xaviers College, Kolkata.

Dr. A. Ghorai, did his doctoral work on the 'Study of point defects in solids and thin films' at IACS, Kol-32 and received Ph.D. degree from Jadavpur University in 1989. Presently Dr. Ghorai is actively engaged in theoretical study of defect and diffusion properties of solids, especially dependence of formation energy of a vacancy in cubic metals using different model potentials, exchange and correlation functions and second order perturbation theory of quantum mechanics. He did several pedagogic experimental project works with undergraduate students. He has published 36 journal papers and six books.