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A Comprehensive Study of Mass Accretion and Atmospheric Effect of Raindrop

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***Abstract:** The mass accretion of a raindrop in different layers of the atmosphere is not dealt with so far. A comprehensive brief study of the motion of raindrops through the atmosphere (i) without mass accretion, (ii) with mass accretion and (iii) finally pressure variation in the atmosphere with altitude using Bernoulli's equation is illustrated. Acquirement of mass from moist air is mass accretion and mass accretion during the motion of raindrop through resistive medium holds an arbitrary power-law equation. Bernoulli's equation when applied to it, the generalized first-order differential equation is reduced to a polynomial equation. Results show a single intersecting point of approximate terminal velocity 1 m/s and mass 10^{-06} mg as illustrated. Terminal velocity is achieved within 25 sec. There is the approximate exponential growth of terminal velocity. An increase in momentum is due to mass accretion during motion. Various conditions of no mass accretion and mass accretion show the same result while for atmospheric effect using Bernoulli's equation the first-order differential equation reduces to a polynomial equation.*

***Keywords:** Air Resistance, Bernoulli's Equation, Cloud Collide, Mass Accretion Rate, Moist Air, Raindrop, Terminal Velocity.*

I. INTRODUCTION

Recent piezoelectric energy harvesting techniques of raindrops [1] and Sokal1's generalized mass accretion formula [2] suggest a need to generalize the dynamics of raindrops which issimple yet most complicated mechanical problems studied since the late 40s of the last century. The constant velocity of falling raindrop is called the terminal velocity and its empirical relation [3] was $v_T = \sqrt{\frac{8gR(\rho_w-\rho_a)}{b\rho_a}}$. Here v_T is in (meter/sec) and *R* the radius of raindrop is in meters. Due to air turbulence, water droplets in clouds collide, hence produce larger droplets. Thus raindrops vary widely in their shapes, sizes and velocities. The drops that make up a significant fraction of rain are less than 1mm in size and their shapes can be well approximated as a sphere.

The dynamics of a falling raindrop in generalized form through moist air in all directions (basically up and down) is governed by the force due to weight of the raindrop $mg\hat{k}$, the buoyant force $m\rho_a/\rho_w\hat{k}$ and the frictional force of the medium, which is proportional to the nth power of velocity $bv^n\hat{k}$. Here ρ_a and ρ_w are the density of the medium and raindrop respectively. We assume $a = g(1-\rho_a/\rho_w)$ and the generalized raindrop equation [3] is

$$m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v} = am\hat{k} - bv^n\hat{k} \qquad \text{-(1)}$$

Here b is the resistive force per unit nth power of velocity and Lynch and Lommatsch [4] took the value of it as $b = 0.15\rho_a A$. The cross-sectional area of raindrop assumes the form $A = 3.3108 \times (2R)^{2.21672}$, with the mass of raindrops as $m = 957.251 \times (2R)^{3.09275}$ where R is the approximate radius. Generally, n=1 case is taken for resistive force proportional to velocity. But there are lot of examples where the case is n>1 and generalized procedure for a solution is to be taken in those cases. In this article, the kinematic behaviour of falling raindrops will be deduced for a variety of cases (i) no mass accretion, (ii) mass accretion and (iii) Bernoulli's equation. In this paper formulation for no mass accretion is dealt with first in section 3. Following which in section 4 mass accretion case is discussed. In section 5 Bernoulli's equation for raindrop motion is formulated by both of us.

II. NO MASS ACCRETION

In case of no mass accretion since $dm/dt = 0$ we get $\frac{dv}{\frac{am}{b}-v^n} = \frac{bdt}{m}$. We use the procedure of rational fraction of integration as

$$\int_0^v \sum_{l=1}^n \frac{s_l dv}{r_l - v} = \int_0^t \frac{bdt}{m} \Rightarrow e^{\frac{bt}{m}} = \prod_{l=1}^n \left(\frac{r_l}{r_l - v}\right)^{s_l} \qquad \text{- (2)}$$

Result of this part was discussed in great detail by Dey and Ghorai [3].

III. MASS ACCRETION

Assuming change in the mass of raindrop linearly with increasing distance and time, Krane [5] suggested a proportionality relation of mass accretion dm/dt with different powers of mass m and velocity v. Adawi [6] suggested $\frac{dm}{dt} = cm^\alpha v$ with $\alpha = 0, \frac{2}{3}$ for zero acceleration case. Partovi and Aston [7] took air resistance to this problem which is proportional to the square of velocity. Sokal [2] generalized Krane's idea by introducing $\frac{dm}{dt} = cm^\alpha v^\beta$ with α and β are (almost) arbitrary exponents $(\alpha;\beta) = (0, \frac{1}{3}, \frac{2}{3}; 0,1)$. The value of the constant c depends on the actual shape of the raindrop and if spherical it is given by

$$c = \frac{\pi R^2}{(\frac{4}{3}\pi R^3 \rho_w)^\alpha} \qquad \text{- (3)}$$

The general solution for mass accretion $m\frac{dv}{dt} + cm^\alpha v^{\beta+1} = am - bv^n$ is rather more complicated.

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For lower values of α, β and n we have done both analytical and computational results for terminal velocity $v_T^{c\alpha\beta}$ which is $cv_T^{c00} + b(v_T^{c00})^n = am$ - (4)
For example, when $\alpha = 0$, $\beta = 0$, $c \neq 0$ and $n = 0$ we have $\frac{dv}{dt} = 0$, therefore, $m = M$ and $M = (\frac{2bm_0 - am_0^2}{a})^{\frac{1}{2}}$. Similar expressions for $\alpha = \frac{1}{3}$, $\beta = 0$, $c \neq 0$ is $cv_T^{c\frac{1}{3}0}M^{\frac{1}{3}} = aM - b(v_T^{c\frac{1}{3}0})^n$ and for $\alpha = \frac{2}{3}$, $\beta = 0$, $c \neq 0$ is $cv_T^{c\frac{2}{3}0}M^{\frac{2}{3}} = aM - b(v_T^{c\frac{2}{3}0})^n$; also for $\beta = 1$ we will arrive at the same type of expressions.

IV. BERNOULLI'S EQUATION

The above modifications seem inadequate because of pressure variation in the atmosphere with altitude and it is something new to incorporate. Here we differentiate the pressure head and energy head of Bernoulli's equation $\frac{v^2}{2} + gz + \frac{P}{\rho_a} =$ constant and combine it with the above equation (1) to arrive at the scalar differential equation

$$v\left[ma - bv^n - \frac{v}{m}\frac{dm}{dt}\right] + g\frac{dz}{dt} + \frac{1}{\rho_a}\frac{dP}{dt} = 0 \quad - (5)$$

Here P is the pressure and z is the altitude. Since $dP = -\rho_a gdz$ we rewrite it using Sokall's idea[2] of $\frac{dm}{dt} - cm^\alpha v^\beta$ as $ma - bv^n - cm^{\alpha-1}v^{\beta+1} = 0$ - (6)
This is a polynomial equation of degree n and is very easy to solve.

V. RESULT AND DISCUSSION

A python code is developed for no mass accretion and it has been shown that terminal velocity is reached almost within ten seconds for a mass of raindrop 0.5 mg and an approximate diameter of 1 mm. The minimum raindrop size below which it may be assumed to be a cloud particle is of mass 10^{-06}mg. All these are illustrated in the velocity-mass graph of Fig. 1a for no mass accretion. It is now necessary to test the applicability of Bernoulli's equation with no mass accretion case first. All these calculations use python programs extensively. Both the graphs in Fig. 1a and Fig. 1b resemble the same nature as predicted earlier [3,5-7]. The plots show close agreement and velocity power up to n=3 perhaps a better choice as shown in Fig. 1b. Here n=0 is not plotted as the terminal velocity is undetermined. The graphs have a single intersecting point.

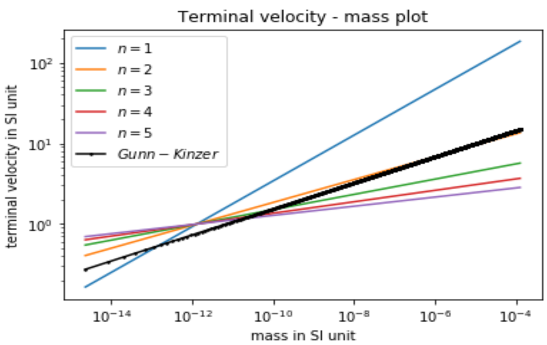

Figure 1a: Terminal velocity-mass graph in SI units for no mass accretion

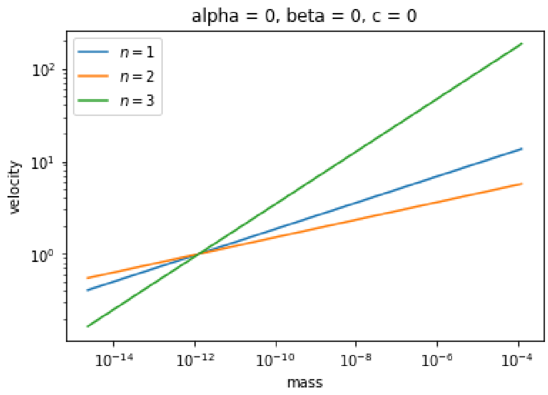

Figure 1b: Terminal velocity-mass graph in SI units for Bernoulli's equation

The next step is to test with and without mass accretion case. The single velocity-time graph of Fig. 2 for without mass accretion case is compared with six graphs of mass accretion case shown in Fig. 3a-3f. The terminal velocity for no mass accretion v_T^{000} is compared with those with mass accretion case $v_T^{c\alpha\beta}$ [$cv_T^{c\alpha\beta}M^{\frac{1}{3}} = aM - b(v_T^{c\alpha\beta})^n$] corresponding to $c \neq 0$, $\alpha = 0, \frac{1}{3}, \frac{2}{3}$ and $\beta = 0,1$. In all these cases terminal velocity is achieved within 25sec. Considering the increase in momentum due to mass accretion with time this increase in $v_T^{c\alpha\beta}$ is almost trivial.

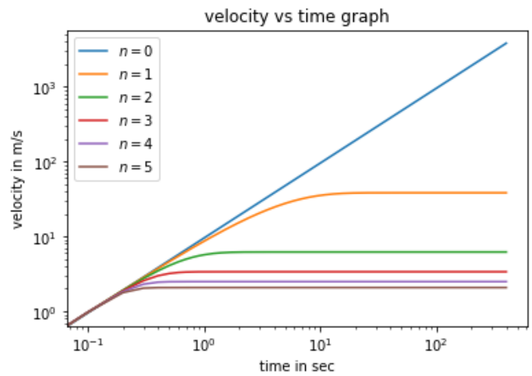

Figure 2: Velocity-time graph for terminal velocity in SI units for no mass accretion

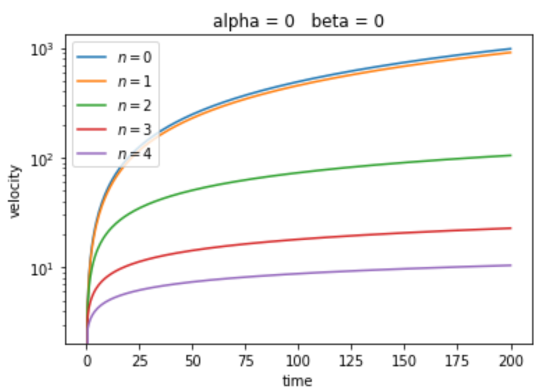

Figure 3a: Velocity-time graphs for terminal velocity in SI units for mass accretion case

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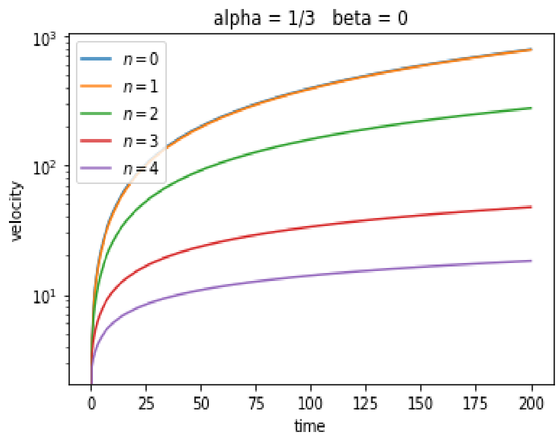

Figure 3b: Velocity-time graphs for terminal velocity in SI units for mass accretion case

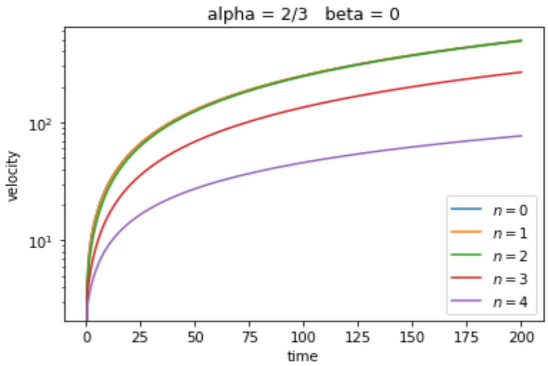

Figure 3c: Velocity-time graphs for terminal velocity in SI units for mass accretion case

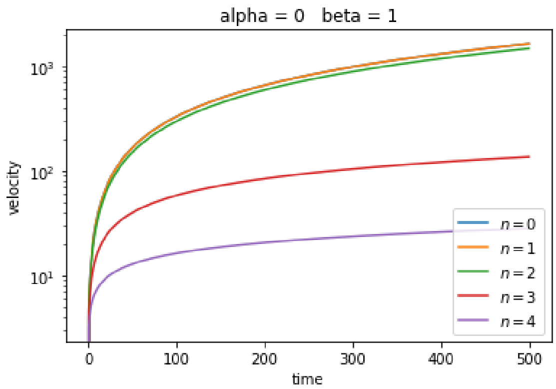

Figure 3d: Velocity-time graphs for terminal velocity in SI units for mass accretion case

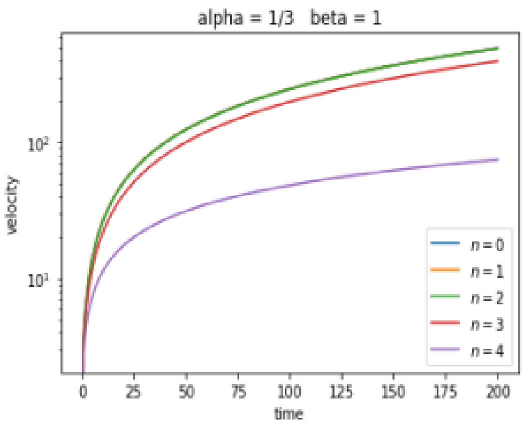

Figure 3e: Velocity-time graphs for terminal velocity in SI units for mass accretion case

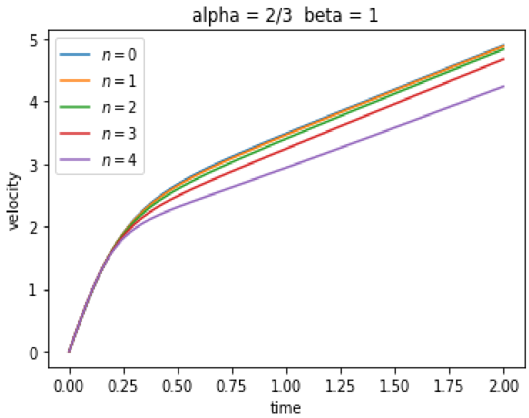

Figure 3f: Velocity-time graphs for terminal velocity in SI units for mass accretion case

In the final step pressure variation in the atmosphere with altitude according to Bernoulli's equation in the polynomial expression for terminal velocity $v_T^{c\alpha\beta}$ is plotted in six plots of Fig. 4a-4f. All these six graphs resemble the previous cases. They speak the truth that they resemble the same nature of terminal velocity achievement with time and mass [8]. Table 1 gives the slope of terminal velocity with mass which was predicted earlier in graphs of Fig.4.

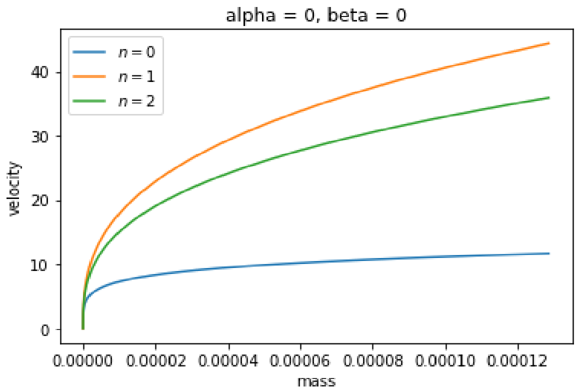

Figure 4a: Velocity-mass graph for Bernoulli's equation in SI units $(\alpha, \beta) = (0, 0)$

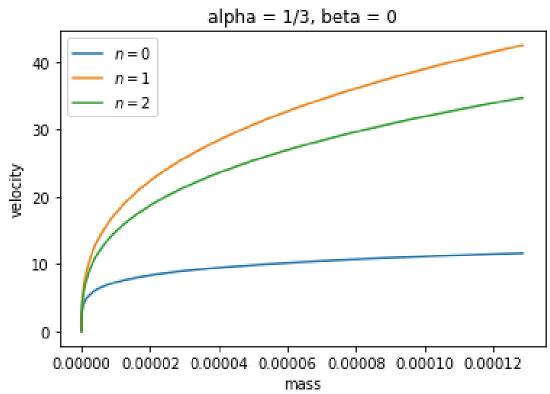

Figure 4b: Velocity-mass graph for Bernoulli's equation in SI units $(\alpha, \beta) = \left(\frac{1}{3}, 0\right)$

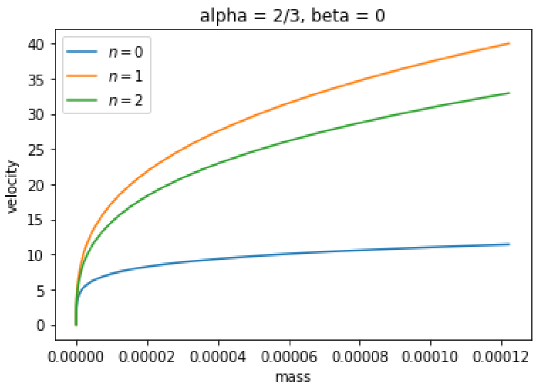

Figure 4c: Velocity-mass graph for Bernoulli's equation in SI units $(\alpha, \beta) = \left(\frac{2}{3}, 0\right)$

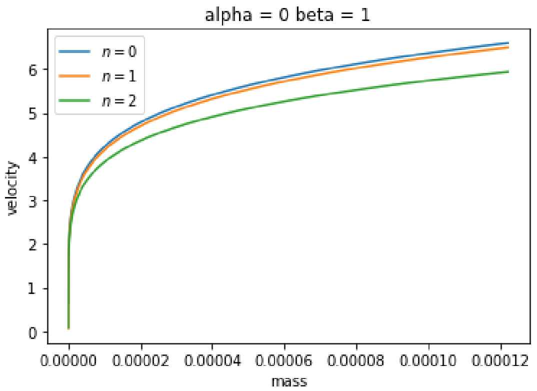

Figure 4d: Velocity-mass graph for Bernoulli's equation in SI units $(\alpha, \beta) = (0, 1)$

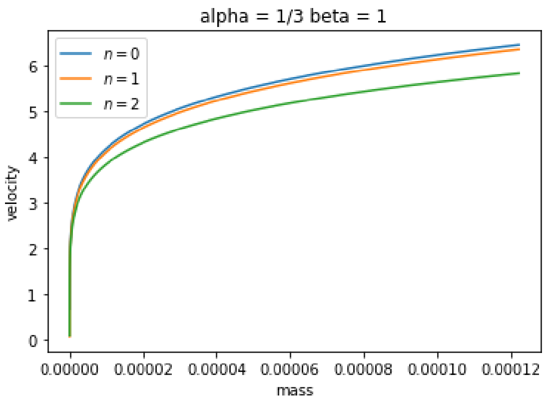

Figure 4e: Velocity-mass graph for Bernoulli's equation in SI units $(\alpha, \beta) = \left(\frac{1}{3}, 1\right)$

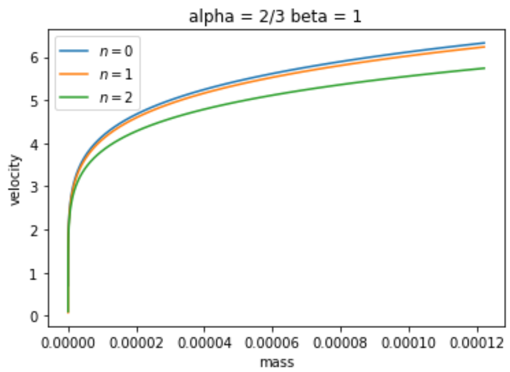

Figure 4f: Velocity-mass graph for Bernoulli's equation in SI units $(\alpha, \beta) = \left(\frac{2}{3}, 1\right)$

Table 1: The slope of variation of terminal velocity $v_T^{c\alpha\beta}$ (SI units) due to mass accretion

$\frac{dv_T^{c\alpha\beta}}{dm}$	n=0	n=1	n=2
(α,β)=(0,0)	1.35×10^5	1.05×10^5	1.80×10^4
(α,β)= (1/3,0)	1.26×10^5	9.94×10^4	1.79×10^4
(α,β)= (2/3,0)	1.18×10^5	9.38×10^4	1.77×10^4
(α,β)= (0,1)	1.05×10^4	1.04×10^4	9.01×10^3
(α,β)= (1/3,1)	9.95×10^3	9.87×10^3	8.66×10^3
(α,β)= (2/3,1)	9.47×10^3	9.40×10^3	8.32×10^3

VI. CONCLUSION

In this work, the comprehensive theoretical idea of various aspects of the motion of raindrops is discussed which is at par with earlier calculations and models. Analytical solution of general mass accretion case of raindrop $m\frac{dv}{dt} = am - bv^{\,n}$ is difficult and so numerical computational process is considered. A complete analytical process may be arrived at in near future.

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Sneha Dey is an undergraduate Physics Honours student of Maulana Azad College, Kol-13. She attended an workshop 'Inspire Camp' organized and sponsored by The Department of Science and Technology, Central Government of India (DST) and DBT held at Jagadis Bose National Science Talent Search, Kolkata, West Bengal, India. She presented a paper and ppt on mechanical properties of matter after working in the workshop.

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